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Moving away from anisotropic lay-ups; modelling of 3D preforms & weaves

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Abstract Six is the number of fibre orientations necessary and sufficient to remove all zero-valued shear moduli from fibre arrays that have cubic symmetry. Of particular interest is a weave, of just four families of flexible fibres, that has equal numbers of equal fibre lengths parallel to the six face diagonals of a cube. The relative positions of fibre lengths in this weave can be visualised as the edges of space-filling equilateral truncated octahedra.

Introduction

Laminates have a propensity to delaminate; the mathematical plane between adjacent plies offers a preferred path for crack propagation, irrespective of the nature of the stress field that gives rise to the elastic strain energy released. This is because the plane between plies is characterised by a specific fracture surface energy significantly lower than that for any other internal surface.

Consider airframe components. The overall stress field experienced by the fuselage is two-dimensional; engine thrust works against drag, and lift works against payload. In a plate subjected to in-plane principal stresses σ_1 and σ_2, respectively making angles ϕ and $\pi/2 - \phi$ with the plane of the starter crack, Griffith [1] predicts the following conditions for fracture.

Taking tensile stress as +ve and $\sigma_2 > \sigma_1$

(i) If $3\sigma_2 + \sigma_1 > 0$, fracture occurs when $\sigma_2 > K$ where K is the strength in uniaxial tension $\phi = 0$, the fracture surface is perpendicular to σ_2

(ii) If $3\sigma_2 + \sigma_1 < 0$, fracture occurs when $(\sigma_2 - \sigma_1)^2 + 8K\,(\sigma_2 + \sigma_1)^2 = 0$

$\cos 2\phi = -1/2(\sigma_2 - \sigma_1)\;/(\sigma_2 + \sigma_1)$ and crack growth from the surface of the most dangerously oriented pre-existing flaw occurs near, but not at, the end of the major axis of that flaw and is on a plane inclined to the directions of principal stress. So, for example, if it has any shear acting on it, the starter crack changes direction and, in a layered material, is free to seek out the plane of weakness

Following Orowan [2], Griffith's two conditions for fracture in two-dimensional stress fields are represented graphically in Fig. 1. If σ_1 and σ_2 are plotted as rectangular co-ordinates, Eq. (ii) is that of a parabola which is concave towards the bisector of negative σ_1 and σ_2

Equation (i) is that to a vertical tangent to this parabola. When all values of σ_1 and σ_2 are considered, those for which $\sigma_1 > \sigma_2$ give rise to fracture when $\sigma_1 > K$ if $3\sigma_1 + \sigma_2 > 0$. This is the equation to a horizontal tangent to the parabola. The resulting fracture locus is shown by the cross-hatched line in Fig. 1. Fracture occurs when the point representing the state of stress crosses the locus towards the cross-hatched side. Taking as example, the quadrant where both principal stresses are compressive, fracture is expected when the two stresses become strongly unequal. For aircraft with engines mounted on wings below the fuselage, the overall stress field is bi-dimensional compression fore, and shear aft, of the wings; the principal stresses become strongly unequal during ''heavy'' landing.

The development of three-dimensional (3D) weaves offers a more reliable way of extending the application of fibre reinforcement to fracture critical structures than does

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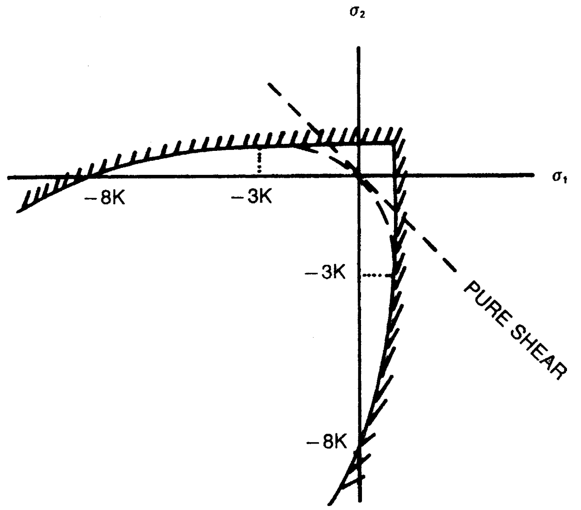


Fig. 1 Failure envelope for the propagation of a Griffith crack in two-dimensional stress fields. After Orowan [2]

the development of laminates, and is the subject of the present investigation.

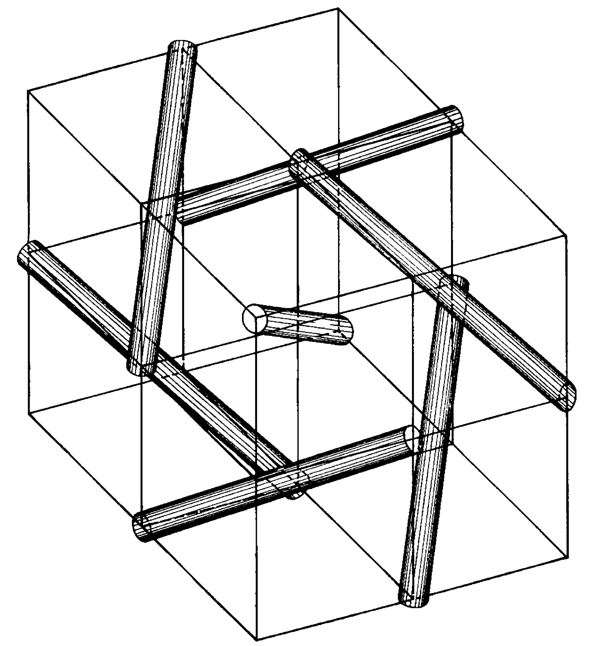


Fig. 2 Three-dimensional cubic symmetry array of identical cylinders parallel to the body diagonals of a cube

Simple cubic symmetry arrays of rigid fibres

There exists an infinite number of cubic symmetry arrangements of identical cylinders. The simplest is that with cylinder axes parallel to the (three) edges of a cube, for which the packing fraction (fibre volume fraction realised when crossing fibres touch) is $\eta = 3\pi/16 \equiv 59\%$. This array is of limited interest as a basis for the manufacture of engineering materials because it is not braced against all orientations of applied shear; a two-dimensional scissors-like deformation about one axis of the mother cube does not tension fibres in any of the three fibre directions.

Figure 2 shows the ''primitive unit cell'', the mother cube, for the array with fibres parallel to the (four) body diagonals of a cube. It is sub-divided into eight octant cubes, within each of which is shown the orientation of one body diagonal. Octant cubes, on opposite sides of the mother cube centre of symmetry, contain parallel fibres. The fibre packing fraction is $\eta = (\pi\sqrt{3})/8 \equiv 68\%$ This array is also not braced against all orientations of applied shear; shear about cube axes does not tension fibres in any of the four fibre directions.

Of more technological interest, since it removes all zero-valued shear moduli, are arrays that have six fibre orientations, specifically the array with fibres parallel to the (six) edges of a regular tetrahedron or, and this is the same array, parallel to the (six) face diagonals of a cube. To visualise best packing for this array, it is instructive to recall Lord Kelvin's theorem that there are three, and only three, regular polyhedra that fill space. In Fig. 2 we used the filling of space by cubes; the other two are rhombic dodecahedra and equilateral truncated octahedra. Figure 3 shows a rhombic dodecahedron with edges parallel to the body diagonals of a cube; it is evident that opposite pairs of faces of the dodecahedron are perpendicular to face diagonals of the cube. In Figs. 4 and 5, the ''face diagonal directions'' array is visualised as fibre lengths threading opposite faces of space-filling rhombic dodecahedra. Figure 4 shows how identically oriented rhombic dodecahedra, threaded by the

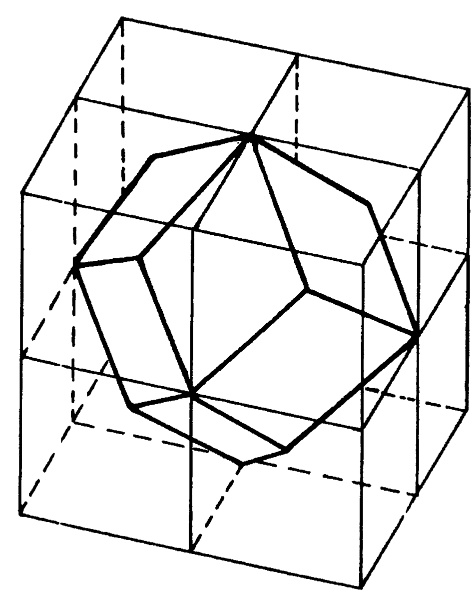
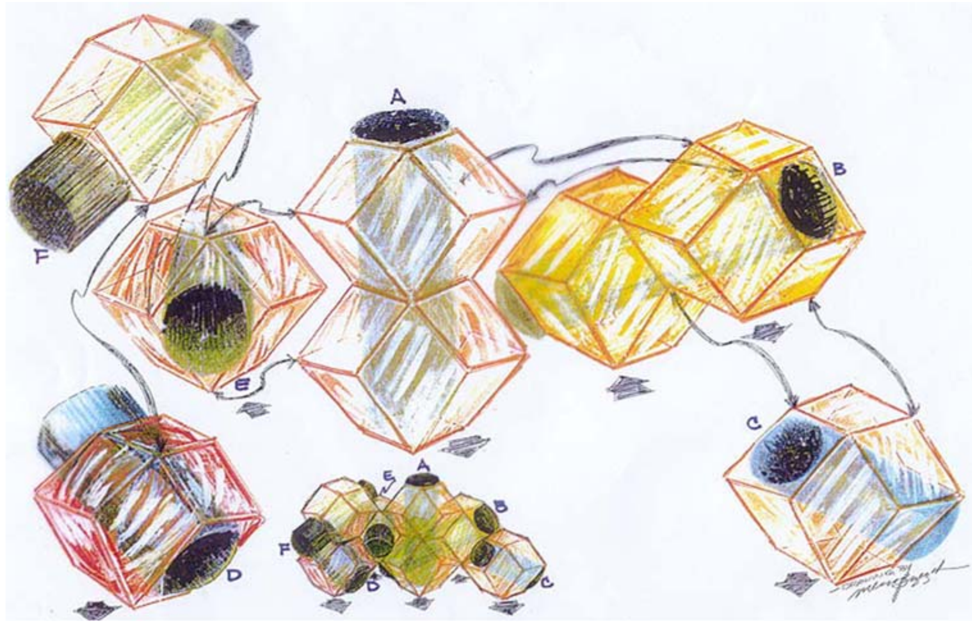


Fig. 3 Rhombic dodecahedron with faces perpendicular to the face diagonal directions of a cube

Fig. 4 Showing how identically oriented rhombic dodecahedra, each containing a fibre parallel to a (different) face diagonal direction of a cube, can be brought together so as to completely fill space



six fibre orientations respectively perpendicular to the six pairs of opposite dodecahedron faces, are assembled to create the array shown in Fig. 5. Neighbouring fibres cross at two kinds of points; fibres which cross at right angles touch at points on common diameters parallel to cube edges, and fibres which cross at 60° do so, without touching, on diameters parallel to cube body diagonals. The packing fraction is $\eta = (\pi\surd 2)/8 \equiv 56\%$.

Super-high fibre volume fractions

By altering the external morphology of fibres, the packing fractions of cubic symmetry arrays can be increased beyond those for identical circular cross-section cylinders. For example, by substituting hexagonal cross-section fibres for fibres of circular cross-section, the packing fraction of the ''cube face diagonals'' array can be increased from $\eta = (\pi\surd 2)/8$ to almost $\eta = 5/8$. However, as can be seen in Fig. 6a, crossing fibres do not touch face-on-face; instead, fibre edges make line contact with fibre faces and, in a real composite, would create classical ''razor edge/flat surface'' linear stress concentrations, Fig. 6b.

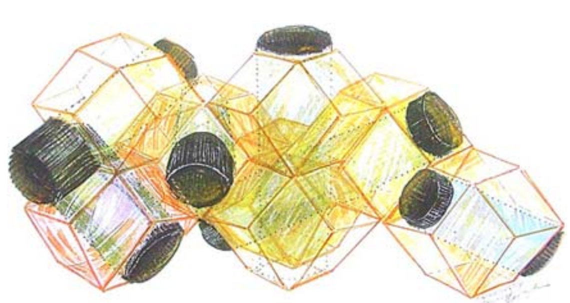


Fig. 5 Space-filling rhombic dodecahedra showing the relative orientations of the six fibre directions parallel to the face diagonals of a cube

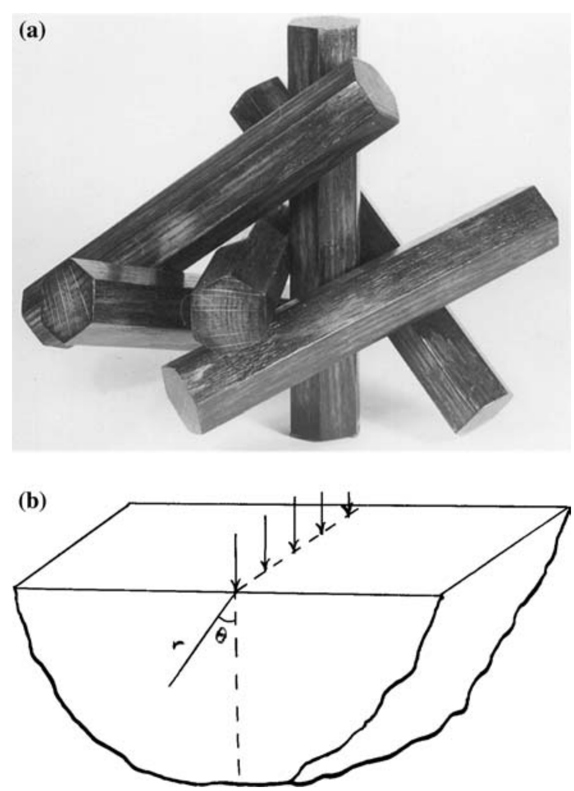


Fig. 6 (**a**) Best-packed cubic symmetry array of hexagonal prisms oriented parallel to the six face diagonals of a cube. (**b**) Schematic of the ''razor edge/flat surface'' contacts between neighbouring prisms in Fig. 6a. $\sigma_{rr} = -(2p/\pi)(\cos\theta/r)$, where p is the load per unit thickness. $\sigma_{\theta\theta} = \sigma_{r\theta} = 0$

Assembly of a ''cube face diagonals'' preform

A miniature high speed (6500 rpm) drilling machine has been used to drill rhombohedral arrays of fibre socket holes, with diameter 140 μm, in 1 mm thick brass plates. A partially assembled ''cube face diagonals'' preform, with four families of fibres located in socket holes in plates clamped to precision-machined orientation blocks, is shown in Fig. 7; note the rhombic dodecahedral form of the enclosed space.

A cubic symmetry weave with no zero-valued shear moduli [3]

Figure 8 shows an equilateral truncated octahedron oriented with all of its edges lying parallel to face diagonals of the cube within which it is drawn. The filling of space by identical equilateral truncated octahedra, Fig. 9, shows how, with just four families of flexible fibres, a 3D weave in which the individual lengths of fibre lie parallel to the six face diagonals of a cube, might be constructed. The overall directions threaded by the four families of fibres are parallel to the four body diagonals of the mother cube. However, along its length, each fibre is segmented by successive 120° bends in such a way that individual segments lie parallel to three different ''face diagonal'' directions, thereby making up a three-fold helix of fibre; fibres from different families cross over at their 120° bends, and it is these fibre cross-overs that hold the weave together.

Cubic symmetry hybrids

When combined, the arrays with fibres parallel to the (three) edges and (four) body diagonals of a cube form an array that has no zero-valued shear moduli. The best

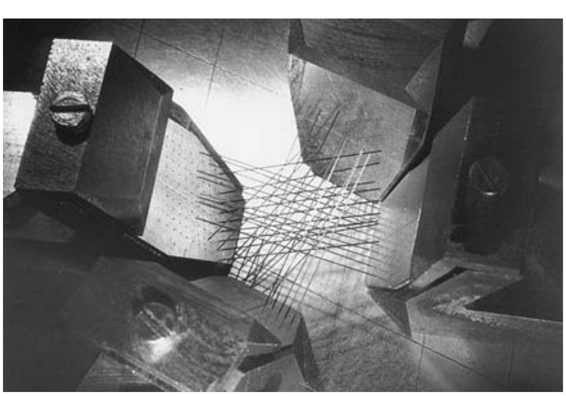


Fig. 7 Jig used to assemble ''face diagonal directions'' arrays of 140 μm diameter silicon carbide fibres

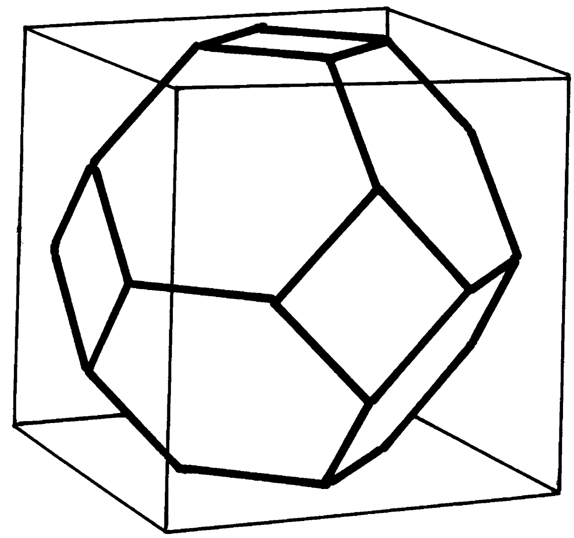


Fig. 8 An equilateral truncated octahedron with all of its edges parallel to face diagonals of a cube

packed (highest fibre volume fraction) spatial positioning of the seven sets of fibres in the combined array can be visualised by reference to Fig. 2. Between the mid-points of parallel edges of the octant cubes, there is room for fibres oriented parallel to the cube edges. There are six of these cube ''edge'' fibre locations per octant cube, each of which is shared with a neighbouring octant cube.

Neighbouring cube ''edge'' and ''body diagonal'' fibres touch along common diameters parallel to face diagonals of the mother cube, and do so at a fibre radius of $a/(4\sqrt{2})$, thereby making the overall packing fraction $\eta = \pi(3 + \sqrt{3})/32 \equiv 46\%$. Hybrids in which one sub-array is assembled from high modulus fibres and the other from high toughness fibres (toughness—the possession of some natural mechanism or mechanisms for rendering cracks non-disastrous, such as slip and twinning in the case of crystalline materials—usually does not go hand-in-hand with high modulus) is a basis for development of ceramic composite materials that are both stiff and tough.

Cube ''edge'' plus cube ''body diagonal'' hybrids, assembled from sub-arrays of fibres that have very different axial tensile modulus, raises the possibility of designing composites with overall elastic isotropy. In crystallography, crystals with cubic symmetry have, in general, three independent elastic constants. In the special case of isotropic crystals, the number of independent elastic constants is reduced to two. The same general and special cases exist for the elasticity of composites based on 3D fibre preforms that have cubic symmetry. The packing fractions for the two sub-arrays of fibres in the hybrid described above are $\eta_e = 3\pi/32$ for that parallel to the edges, and $\eta_d = (\pi\sqrt{3})/32$ for that parallel to the body diagonals of the mother cube, making the ratio of sub-array

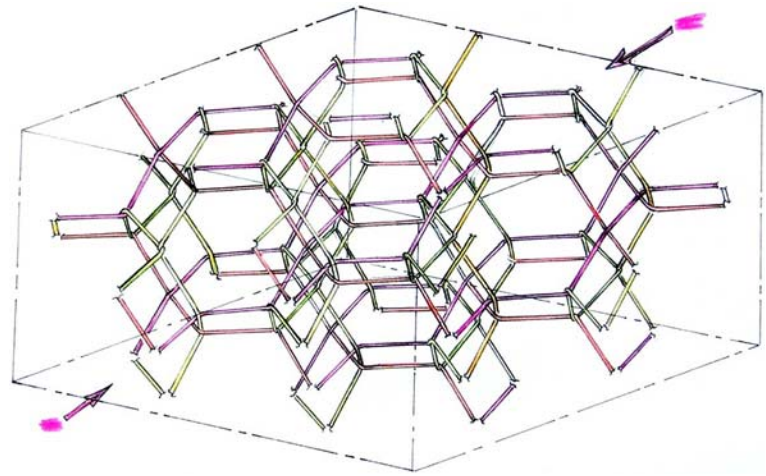


Fig. 9 Space-filling identically oriented equilateral truncated octahedra delineated by four sets of continuous fibres whose lengths lie along the face diagonal directions of a cube

packing fractions $\eta_e/\eta_d = \sqrt{3}$. For combined sub-arrays assembled from a single fibre species to behave isotropically, this ratio needs to be $\eta_e/\eta_d = 2/3$ [4], i.e. most of the ''cube edge'' locations would have to be empty. However, by assembling a hybrid with elastically stronger fibres making up the ''body diagonals'' sub-lattice, it may be possible to achieve elastic isotropy using fibre species that have the same area of cross-section.

Preforms, comprising ''Saphikon'' single crystal sapphire whiskers (hexagonal cross-section with nominal hexagon edge length 50 μm) in the four ''body diagonal'' directions and ''BP Chemicals'' 140 μm diameter silicon carbide whiskers in the three cube ''edge'' directions, have been assembled by first constructing the silicon carbide sub-array from rafts of identically spaced parallel fibres. To make the rafts, ten 50 mm lengths of silicon carbide fibre, each with two 10 mm lengths of 35 gauge hyperdermic needle threaded onto it, one onto each of its ends, and separated from its two neighbours by two pairs of empty 10 mm lengths of the same tubing, have been sandwiched, top and bottom, between 18 mm square 200 μm thick glass cover slips. A photograph of one of these rafts is reproduced in Fig. 10. By laying down twenty such rafts, with alternate rafts at right angles to each other and with a cover slip spacing apart the cover slips bonded to the ends of neighbouring parallel rafts, one hundred fibres by one hundred fibres 3D arrays of two of the three cube ''edge'' oriented fibres have been assembled as

Fig. 10 Raft of ten 140 μm diameter silicon carbide whiskers

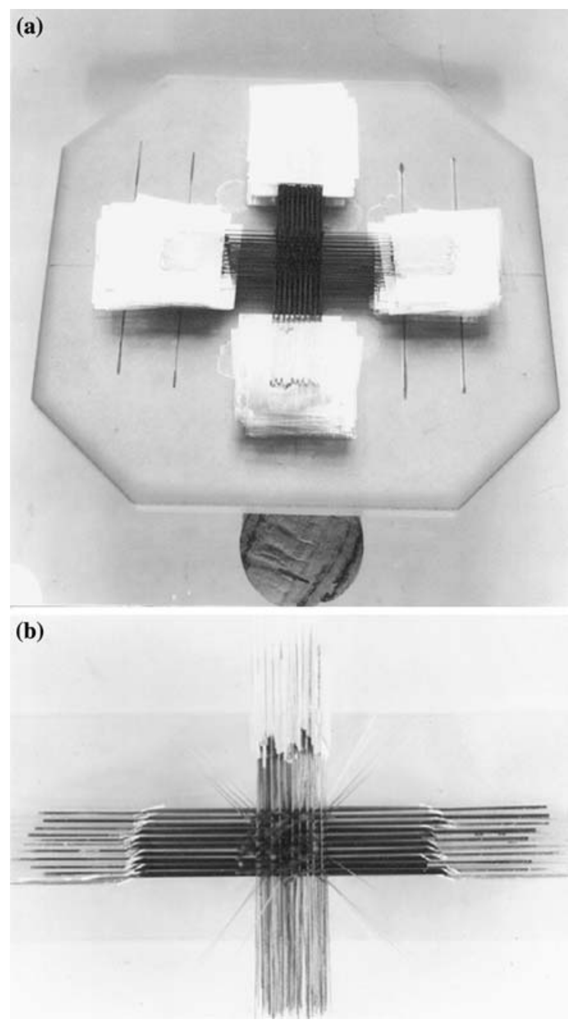


Fig. 11 (**a**) Twenty high stack of alternately perpendicular rafts of ten 140 μm diameter silicon carbide whiskers. (**b**) Single crystal sapphire whiskers inserted into the channels parallel to all four body diagonals of the stack shown in Fig. 11a

shown in Fig. 11a. One hundred fibres oriented parallel to the third cube ''edge'' direction were then threaded into the channels left by the existing two sets. Using an equilateral truncated octahedron model to determine the locations relative to individual cube ''edge'' fibres, ''body diagonal'' sapphire fibres were inserted into the latter. A seven fibre orientations hybrid preform so assembled is shown in Fig. 11b.

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